Radiative $\Delta \to N\gamma$ decay in light cone QCD

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The $g_{\Delta N\gamma}$ coupling for the $\Delta \to N\gamma$ decay is calculated in the framework of the light cone QCD sum rules and is found to be $g_{\Delta N\gamma}=(1.6\pm 0.2)$ GeV^{-1}. Using this value of $g_{\Delta N\gamma}$ we estimate the branching ratio of the $\Delta^+ \to N\gamma$ decay, which is in a very good agreement with the experimental result. [S0556-2821(99)02023-8]

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I. INTRODUCTION

The extraction of the fundamental parameters of hadrons from experimental results requires some information about physics at large distance. Unfortunately such information cannot be achieved from the first principles of a fundamental theory. Indeed QCD, which is believed to be a candidate of such an underlying theory of strong interactions, has very complicated infrared behavior which makes it impossible to calculate the properties of the hadrons starting from a fundamental QCD Lagrangian. Therefore for the determination of the parameters of hadrons a reliable nonperturbative approach is needed. Among all nonperturbative approaches, QCD sum rules, which were originally proposed by Shifman, Vainshtein, and Zakharov [1] and adopted or extended in many works [2–6], are particularly powerful in studying the properties of low-lying hadrons. In the traditional QCD sum rules method [1] the nonperturbative approach is taken into account through various condensates in the nontrivial QCD vacuum.

In this work we employ an alternative approach to the traditional sum rules, namely, light cone QCD sum rules, to study the $\Delta \to N\gamma$ decay coupling constant. Light cone sum rules is based on the operator product expansion on the light cone, which is an expansion over the twists of the operators rather than dimensions in the traditional sum rules. The main contribution comes from the lowest twist operator. The matrix elements of the nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadronic wave function (more about application of light cone QCD sum rules can be found in Refs. [7–16], and references therein).

In general $\Delta \to N\gamma$ decay is described by the electric quadrapole $E2$ and magnetic dipole $M1$ transition amplitudes. However, it is well known that the electric quadrapole amplitude is very small compared to that of the magnetic dipole amplitude (see Ref. [17], and references therein). Therefore in this work we consider only the magnetic dipole contribution. The coupling constant $g_{\Delta N\gamma}$ is involved in phenomenological models in investigation of the many reactions of the strong and electromagnetic interactions and it is expected to be measured more precisely in the pion photo production experiments at TJNAL (former CEBAF).

The paper is organized as follows. In Sec. II the light cone QCD sum rules for the radiative $\Delta \to N\gamma$ is presented. Section III is devoted to the numerical analysis and discussion of the results.

II. THE LIGHT CONE QCD SUM RULES FOR $\Delta \to N\gamma$ DECAY CONSTANT

In studying the $\Delta \to N\gamma$ decay constant we first introduce the interpolating currents for the Δ and N baryons [2]:

$$\eta^{\mu}_{\Delta^+}=\frac{1}{\sqrt{3}}\epsilon^{abc}[(u_a^T\mathcal{C}\gamma^{\mu}u_b)d_c+2(u_a^T\mathcal{C}\gamma^{\mu}d_b)u_c],$$

$$\eta_N=\epsilon^{abc}(u_a^T\mathcal{C}\gamma_{\mu}u_b)\gamma_5\gamma^{\mu}d_c, \qquad (1)$$

where u,d are up and down quark fields, respectively, and $\mathcal{C}$ is the charge conjugation operator; a, b, and c are the color indices. Note that this choice of nucleon interpolating currents is not unique and other choices can be used (see, for example, Ref. [3]).

The overlap amplitudes of the interpolating currents with the baryons are defined as

$$\langle 0|\eta_N|N\rangle=\lambda_N u_N,$$

$$\langle 0|\eta^{\mu}_{\Delta}|\Delta\rangle=\frac{1}{\sqrt{3}}\lambda_{\Delta}u^{\mu}_{\Delta}, \qquad (2)$$

where u^{μ}_{Δ} is the Rarita-Schwinger spinor. The coupling constant $g_{\Delta N\gamma}$ for the $\Delta \to N\gamma$ decay is defined as follows:

$$\langle N\gamma|\Delta\rangle=ieg_{\Delta N\gamma}\epsilon_{\mu\nu\alpha\beta}\overline{u_N}\gamma^{\nu}q^{\alpha}\varepsilon^{\beta}u^{\mu}_{\Delta}, \qquad (3)$$

where ε_{μ} and q_{μ} are the polarization vector and momentum of the photon, respectively, e is the electric charge.

According to the QCD sum rules ideology, the quantitative estimates of the $g_{\Delta N\gamma}$ coupling constant can be obtained by equating the two different representations of a suitable correlator, written in terms of hadrons and quark-gluon language. For this purpose we start our analysis by considering the following correlator:

$$\Pi(p,q)=\int d^4x\, e^{ipx}\langle\gamma(q)|\mathrm{T}\{\eta_{\Delta}(0)\overline{\eta}_N(x)\}|0\rangle. \qquad (4)$$

Saturating Eq. (4) by Δ and nucleons and using Eqs. (2) and (3), we get for the phenomenological part of the correlator

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$$ieg_{\Delta N\gamma}\frac{\lambda_N\lambda_\Delta}{\sqrt{3}(p^2-m_N^2)[(p+q)^2-m_\Delta^2]}(\not p[\varepsilon_\mu(pq)-(\varepsilon p)q_\mu])$$
$$+\text{other structures}. \tag{5}$$

The main theoretical problem being the calculation of Eq. (4) in QCD. This problem can be solved in the deep Euclidean region where p^2 and $(p+q)^2$ are negative and large. After lengthy calculations, at quark level we have obtained the following expression for the correlator function

$$\Pi(p,q)=-\frac{1}{4\sqrt{3}}\int d^4x\, e^{ipx}\{-4\langle\gamma(q)|\bar{d}\gamma_5\gamma_\varphi d[\gamma_5\gamma_\varphi\gamma_\rho\gamma_5\,\mathrm{Tr}(\mathcal{S}\gamma_\rho\mathcal{S}'\gamma_\mu)+2\mathcal{S}\gamma_\rho\mathcal{S}'\gamma_\mu\gamma_5\gamma_\varphi\gamma_\rho\gamma_5]|0\rangle$$
$$+2\langle\gamma(q)|\bar{d}\sigma_{\alpha\beta}[\sigma_{\alpha\beta}\gamma_\rho\gamma_5\,\mathrm{Tr}(\mathcal{S}\gamma_\rho\mathcal{S}'\gamma_\mu)+2\mathcal{S}\gamma_\rho\mathcal{S}'\gamma_\mu\sigma_{\alpha\beta}\gamma_\rho\gamma_5]|0\rangle$$
$$-4\langle\gamma(q)|\bar{u}\gamma_5\gamma_\varphi u[2\mathcal{S}\gamma_\rho\gamma_5\,\mathrm{Tr}(\gamma_5\gamma_\varphi\gamma_\rho\mathcal{S}'\gamma_\mu)-2\mathcal{S}\gamma_\rho\gamma_5\gamma_\varphi\gamma_\mu\mathcal{S}\gamma_\rho\gamma_5+2\gamma_5\gamma_\varphi\gamma_\rho\mathcal{S}'\gamma_\mu\mathcal{S}\gamma_\rho\gamma_5]|0\rangle$$
$$+2\langle\gamma(q)|\bar{u}\sigma_{\alpha\beta}u[2\mathcal{S}\gamma_\rho\gamma_5\,\mathrm{Tr}(\sigma_{\alpha\beta}\gamma_\rho\mathcal{S}'\gamma_\mu)+2\mathcal{S}\gamma_\rho\sigma_{\alpha\beta}\gamma_\mu\mathcal{S}\gamma_\rho\gamma_5+2\sigma_{\alpha\beta}\gamma_\rho\mathcal{S}'\gamma_\mu\mathcal{S}\gamma_\rho\gamma_5]|0\rangle\}, \tag{6}$$

where $\mathcal{S}'\equiv\mathcal{CSC}=-\mathcal{CSC}^{-1}$ and $i\mathcal{S}(x)$ is the full light quark propagator with both perturbative and nonperturbative contributions

$$i\mathcal{S}(x,0)=\langle 0|\mathrm{T}\{\bar{q}(x)q(0)\}|0\rangle$$
$$=i\frac{\not x}{2\pi^2x^4}-\frac{\langle\bar{q}q\rangle}{12}-\frac{x^2}{192}m_0^2\langle\bar{q}q\rangle$$
$$-ig_s\frac{1}{16\pi^2}\int_0^1 du\left\{\frac{\not x}{x^2}\sigma_{\alpha\beta}G^{\alpha\beta}(ux)\right.$$
$$\left.-4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu\right\}+\cdots. \tag{7}$$

It follows from Eq. (6) that, in order to calculate the correlator function Π in QCD, the matrix elements $\langle\gamma(q)|\bar{q}\gamma_\alpha\gamma_5 q|0\rangle$ and $\langle\gamma(q)|\bar{q}\sigma_{\alpha\beta}q|0\rangle$ are needed. These matrix elements are defined in terms of the photon wave functions as follows [18–20]:

$$\langle\gamma(q)|\bar{q}\gamma_\alpha\gamma_5 q|0\rangle=\frac{f}{4}e_q e\,\epsilon_{\alpha\beta\rho\sigma}\varepsilon^\beta q^\rho x^\sigma\int_0^1 du\, e^{iuqx}\psi(u),$$

$$\langle\gamma(q)|\bar{q}\sigma_{\alpha\beta}q|0\rangle$$
$$=ie_q e\langle\bar{q}q\rangle\int_0^1 du\, e^{iuqx}$$
$$\times\{(\varepsilon_\alpha q_\beta-\varepsilon_\beta q_\alpha)[\chi\phi(u)+x^2[g_1(u)-g_2(u)]]$$
$$+[qx(\varepsilon_\alpha x_\beta-\varepsilon_\beta x_\alpha)+\varepsilon x(x_\alpha q_\beta-x_\beta q_\alpha)]g_2(u)\}, \tag{8}$$

where the parameter χ is the magnetic susceptibility of the quark condensate and e_q is the quark charge. In further analysis the path ordered gauge factor

$$\mathcal{P}exp\left(\int_0^1 du\, x^\mu A_\mu(ux)\right),$$

is omitted since in the fixed point gauge $x^\mu A_\mu=0$. The functions $\phi(u)$ and $\psi(u)$ in Eq. (8) are the leading twist photon functions, while $g_1(u)$ and $g_2(u)$ are the twist-4 functions. Using Eqs. (6), (7), and (8), and performing Fourier transform for the structure $[\varepsilon_\mu(qp)-(\varepsilon p)q_\mu]$, we get the following result:

$$\Pi=-\frac{i}{\pi^2}\frac{\langle\bar{q}q\rangle}{4\sqrt{3}}\int_0^1 du\left\{(e_u-e_d)\left[\frac{1}{3}\chi\phi(u)\ln(-P^2)\right.\right.$$
$$\left.-[4g_1(u)+2g_2(u)]\frac{1}{P^2}+\frac{2\pi^2}{3P^4}f\psi(u)+\frac{\pi^2}{3P^6}fm_0^2\psi\right]$$
$$+\frac{1}{3P^6}g_2(u)\langle g^2G^2\rangle(-7e_u+3e_d)$$
$$\left.-\frac{1}{12}e_d\langle g^2G^2\rangle\chi\phi(u)\frac{1}{P^4}-\frac{2}{3}g_1(u)e_d\langle g^2G^2\rangle\frac{1}{P^6}\right\}, \tag{9}$$

where $P=p+qu$.

As has been noted already, the QCD sum rule is obtained as usual by equating the hadronic representation of the correlator (4) with the result of the QCD calculation. In order to take into account the contributions of the higher states we invoke the quark-hadron duality prescription, i.e., above certain thresholds in s_1 and s_2, the double spectral density $\rho(s_1,s_2)$ for the higher states and continuum coincides with the spectral density calculated in QCD.

After performing double Borel transformation with respect to the variables p^2 and $(p+q)^2$ in Eqs. (4) and (9) to suppress the higher states, we finally get the following sum rules for the $\Delta N\gamma$ coupling constant:

$$g_{\Delta N\gamma}\lambda_\Delta\lambda_N=-\frac{1}{4}e^{m^2/M^2}\frac{\langle\bar{q}q\rangle}{\pi^2}\Bigg\{(e_u-e_d)\Bigg[-\frac{1}{3}\chi\phi(u_0)M^4f_1(s_0/M^2)+[4g_1(u_0)+2g_2(u_0)]M^2f_0(s_0/M^2)+\frac{2\pi^2}{3}f\psi(u_0)\left(1-\frac{m_0^2}{4M^2}\right)\Bigg]+\left[\frac{1}{6M^2}(-7e_u+3e_d)g_2(u_0)+\frac{1}{3M^2}g_1(u_0)-\frac{e_d}{12}\chi\phi(u_0)\right]\langle g^2G^2\rangle\Bigg\}, \quad (10)$$

where the function

$$f_n(x)=1-e^{-x}\sum_{k=0}^{n}\frac{(x)^k}{k!},$$

is the factor used to subtract the continuum, s_0 is the continuum threshold and

$$u_0=\frac{M_2^2}{M_1^2+M_2^2}, \quad M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2},$$

where M_1^2 and M_2^2 are the Borel parameters. Since masses of the proton, and Δ^+ are very close to each other, we can choose M_1^2 and M_2^2 to be equal to each other, i.e., $M_1^2=M_2^2=2M^2$, from which it follows that $u_0=1/2$.

III. NUMERICAL RESULTS

It follows from Eq. (10) that the main input parameters of the sum rules are photon wave functions. It was shown in Refs. [7,8] that the leading photon wave functions receive only small corrections from the higher conformal spin, so they do not deviate much from the asymptotic form. Following Refs. [18–20], we shall use for the photon wave function

$$\phi(u)=6u\bar{u},$$

$$\psi(u)=1,$$

$$g_1(u)=-\frac{1}{8}\bar{u}(3-u),$$

$$g_2(u)=-\frac{1}{4}\bar{u}^2,$$

where $\bar{u}=1-u$. The values of the other input parameters we have used: $f=0.028$ GeV^2 and $\chi=-4.4$ GeV^2 [21] at the scale $\mu=1$ GeV, $\langle g^2G^2\rangle=0.474$ GeV^2 and for the continuum threshold s_0 we have chosen two different values, i.e., $s_0=2.8$ GeV^2 and $s_0=3$ GeV^2. Having fixed the input parameters, one must find the range of values of M^2 for which the sum rule (10) is reliable. The lowest value of M^2 is usually determined by the condition that the terms proportional to the highest inverse power of the Borel parameter stay reasonably small. The upper limit is determined by demanding that the continuum and and higher state contribution does not get too large, say less than 30% of the leading twist contributions. Both conditions are satisfied in the interval 1 GeV$^2\leqslant M^2\leqslant 1.5$ GeV^2. The dependence of the right side of Eq. (10) on M^2 is shown in Fig. 1. It follows from this figure that in the abovementioned working region of M^2 the sum rule is quite stable. From this figure one can directly predict that

$$g_{\Delta N\gamma}\lambda_\Delta\lambda_N=(0.0020\pm0.0004)\ \text{GeV}^5. \quad (11)$$

Dividing this product of couplings by the residues of proton and Δ currents λ_N and λ_Δ, that were calculated in the analysis of mass sum rules for baryons [2] (see also Refs. [3] and [4])

$$g_{\Delta N\gamma}=(1.6\pm0.2)\ \text{GeV}^{-1}. \quad (12)$$

This prediction of the coupling constant permits us to estimate the width of the $\Delta\to N\gamma$ decay. Using the matrix element for the $\Delta\to N\gamma$ transition [see Eq. (3)], we get for the decay width

$$\Gamma=\frac{\alpha}{4m_\Delta^3}g_{\Delta N\gamma}^2(m_\Delta^2-m_N^2)^3\left[1+\frac{1}{3m_\Delta^2}(m_\Delta-m_N)^2\right], \quad (13)$$

where α is the fine structure constant and m_Δ and m_N are the masses of Δ and nucleons, respectively. Using the predicted value of $g_{\Delta N\gamma}$ in Eq. (12), the result we get for the decay width is

$$\Gamma\simeq0.65(1\pm0.25)\ \text{MeV},$$

and for the branching ratio of this channel we have (for the total decay width we have used $\Gamma_{\text{tot}}=113$ MeV [22])

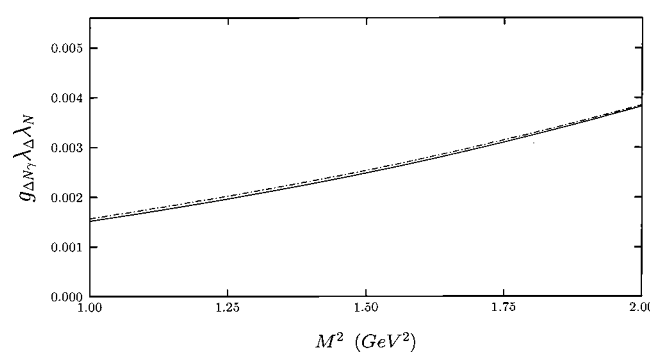

FIG. 1. The dependence of $g_{\Delta N\gamma}\lambda_\Delta\lambda_N$ on the Borel parameter M^2. In this figure the solid and dash-dotted lines correspond to the threshold values $s_0=2.8$ GeV^2 and $s_0=3.0$ GeV^2, respectively.

$$\mathcal{B}(\Delta \rightarrow N\gamma) = \frac{\Gamma}{\Gamma_{\text{tot}}} = 0.0058.$$

This prediction is in a very good agreement with the experimental results, i.e., $0.52\% \leqslant \mathcal{B} \leqslant 0.60\%$ [22].

In summary, we have calculated $\Delta N\gamma$ coupling using the light cone QCD sum rules. Our prediction on the branching ratio is in good agreement with the experimental results.

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